



Calday Maths Department

Preparation for Studying Mathematics at A Level

There is a big jump in difficulty level from GCSE mathematics to A Level mathematics, so in order to have the best start to the course it is advised that you do some work in preparation for it.

Many students find A Level Maths a challenge compared with GCSE, this is a familiar issue, so if you do have a rocky start you are not alone! The maths department is here to help you with the transition to A Level and throughout the course.

When you start the course, much of the content of GCSE will be presumed knowledge and having a good knowledge of the algebra content of GCSE is essential for success at A Level. Before the AS course starts in September we advise you to complete the following exercises to give you valuable practice of this presumed knowledge, a lot of which will be revisited but only briefly.

These practice exercises will also give you an idea of the level of competence that you need to rise to during the first half term of the course.

Further to this work you may want to look at the A Level content on MyMaths (login Calday password circle). If you go to the A Level tab and "Bridging from GCSE", you can do selections of those online tasks.

All students are required to have a Casio FX991 for the A Level course. These are available from the online school shop.

Contents:

Page 2 Exercises

Page 14 Solutions



MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

Exercise 1.1

- 1 Find the values of the letters p , q and r that make the following pairs of expressions always equal.

$$(a) \quad \frac{1}{7}x = \frac{x}{p} \quad (b) \quad \frac{1}{5}(2x+3) = \frac{(2x+3)}{q} \quad (c) \quad \frac{3}{10}(2-7x) = \frac{3(2-7x)}{r}$$

- 2 Solve the following equations.

$$(a) \quad \frac{60}{x+4} = 12 \quad (b) \quad \frac{35}{2x-3} = 5 \quad (c) \quad \frac{20}{6-x} = \frac{1}{2}$$

- 3 Make $\cos C$ the subject of the formula $c^2 = a^2 + b^2 - 2ab \cos C$.

- 4 (a) Multiply $\frac{x+5}{4}$ by 8. (b) Multiply $(x+2) \div 3$ by 12.
 (c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

- 5 Solve the following equations.

$$(a) \quad \frac{3}{4}(2x+3) = \frac{5}{8}(x-2) \quad (b) \quad \frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$$

$$(c) \quad \frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$$

- 6 Make x the subject of the following equations.

$$(a) \quad \frac{a}{b}(cx+d) = x+2 \quad (b) \quad \frac{a}{b}(cx+d) = \frac{2a}{b^2}(x+2d)$$

- 7 Simplify the following as far as possible.

$$(a) \quad \frac{a+a+a+a+a}{5} \quad (b) \quad \frac{b+b+b+b}{b}$$

$$(c) \quad \frac{c \times c \times c \times c \times c}{c} \quad (d) \quad \frac{d \times d \times d \times d}{4}$$

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

Exercise 1.2

1 Work out the following. Answers may be left as improper fractions.

(a) $\frac{4}{7} \times 5$	(b) $\frac{5}{12} \times 3$	(c) $\frac{7}{9} \times 2$	(d) $\frac{4}{15} \times 3$
(e) $\frac{8}{11} \div 4$	(f) $\frac{8}{11} \div 3$	(g) $\frac{6}{7} \div 3$	(h) $\frac{6}{7} \div 5$
(i) $\frac{3x}{y} \times x$	(j) $\frac{3x}{y^2} \times y$	(k) $\frac{5x^3}{4y} \div x$	(l) $\frac{5x^2}{6y} \div y$
(m) $\frac{5x^3}{2y} \times 3x$	(n) $\frac{3y^4}{4x^2z} \times 2x$	(o) $\frac{6x^2y^3}{5z} \div 2xy$	(p) $\frac{5a^2}{6x^3z^2} \div 2y$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$	(b) $V = \frac{4}{3}\pi x^3$	(c) $\frac{1}{2}(u + v) = tx$	(d) $W = \frac{2}{3}\pi x^2 h$
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3 Simplify the following compound fractions.

(a) $\frac{\frac{1}{x} + 1}{\frac{1}{x} + 3}$	(b) $\frac{\frac{2}{x} + 1}{\frac{3}{x} - 1}$	(c) $\frac{\frac{1}{x+1} + 2}{\frac{1}{x+1} + 1}$
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4 Write as single fractions.

(a) $\frac{2}{x-1} + \frac{1}{x+3}$	(b) $\frac{2}{x-3} - \frac{1}{x+2}$	(c) $\frac{1}{2x-1} - \frac{1}{3x+2}$	(d) $\frac{3}{x+2} + 1$
(e) $2 - \frac{1}{x-1}$	(f) $\frac{2x}{x+1} - 3$	(g) $\frac{3}{4(2x-1)} - \frac{1}{4x^2-1}$	

5 Write as single fractions.

(a) $\frac{x+1}{\sqrt{x}} + \sqrt{x}$	(b) $\frac{2x}{\sqrt{x+3}} + \sqrt{x+3}$	(c) $\frac{x}{\sqrt[3]{x-2}} + \sqrt[3]{(x-2)^2}$
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6 Write the following in the form $1 + \frac{a}{x+b}$.

(a) $\frac{x+1}{x-5}$	(b) $\frac{x+3}{x+1}$	(c) $\frac{x+2}{x+5}$	(d) $\frac{x-6}{x-2}$
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**MATHEMATICS A
AND MATHEMATICS B (MEI)****Student Guide****Exercise 1.3**

1 Write without brackets.

(a) $(x + 5)^2$ (b) $(x - 4)^2$ (c) $(2x + 1)^2$

(d) $(3x - 2)^2$ (e) $(x + 2)(x - 2)$ (f) $(3x + 4)(3x - 4)$

2 Simplify the following equations into the form $ax + by + c = 0$.

(a) $(x + 3)^2 + (y + 4)^2 = (x - 2)^2 + (y - 1)^2$

(b) $(x + 5)^2 + (y + 2)^2 = (x - 5)^2 + (y - 2)^2$

(c) $(2x + 1)^2 + (y - 3)^2 = (2x + 3)^2 + (y + 1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2 + 4}$ (b) $\sqrt{x^2 - 4x + 4}$ (c) $\sqrt{x^2 - 1}$

(d) $\sqrt{x^2 + 9x}$ (e) $\sqrt{x^2 - y^2}$ (f) $\sqrt{x^2 + 2xy + y^2}$

4 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 + 2x - 4$

(d) $x^2 - 4x - 3$ (e) $x^2 - 3x + 2$ (f) $x^2 - 5x - 6$

5 Write the following in the form $a(x + b)^2 + c$.

(a) $3x^2 + 6x + 7$ (b) $5x^2 - 20x + 17$ (c) $2x^2 + 10x + 13$

**MATHEMATICS A
AND MATHEMATICS B (MEI)****Student Guide**

6 Write the following in the form $(ax + b)^2 + c$.

(a) $4x^2 + 12x + 14$

(b) $9x^2 - 12x - 1$

(c) $16x^2 + 40x + 22$

7 Factorise as fully as possible.

(a) $x^2 - 25$

(b) $4x^2 - 36$

(c) $4x^2 - 9y^4$

(d) $3x^2 - 7x + 2$

(e) $3x^2 - 5x + 2$

(f) $6x^2 - 5x - 6$

(g) $8x^2 - 2x - 15$

8 Multiply out and simplify.

(a) $\left(x + \frac{1}{x}\right)^2$

(b) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

(c) $\left(x + \frac{2}{x}\right)\left(x - \frac{3}{x}\right)$

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

Exercise 1.4

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\frac{4+6x}{2}$ (d) $\frac{4 \times 6x}{2}$ (e) $\frac{3x+xy}{x}$

(f) $\frac{3x \times xy}{x}$ (g) $\frac{4x+10y}{8x+6y}$ (h) $\frac{3x-6y}{9x-3y}$

(i) $\frac{4x+9y}{2x+3y}$ (j) $\frac{4x+6y}{6x+9y}$ (k) $\frac{5xy+6y^2}{10x+12y}$

(l) $\frac{3x^2+4y^2}{6x^2-8y^2}$ (m) $\frac{x-3}{3-x}$ (n) $\frac{x^2-2xy-y^2}{y^2+2xy-x^2}$

2 Make x the subject of the following formulae.

(a) $\frac{ax}{b} = \frac{py}{qz}$ (b) $\frac{3\pi ax}{b} = \frac{4y^2}{qz}$

3 Simplify the following.

(a) $\frac{2\pi x}{ab} \div \frac{1}{3}\pi r^3$ (b) $\frac{2\pi h^2}{rb} \div \frac{4}{3}\pi hr^2$

4 Simplify into a single factorised expression.

(a) $(x-3)^2 + 5(x-3)^3$ (b) $4x(2x+1)^3 + 5(2x+1)^4$

(c) $\frac{1}{2}k(k+1) + (k+1)$ (d) $\frac{1}{6}k(k+1)(2k+1) + (k+1)^2$

5 Simplify as far as possible.

(a) $\frac{x^2+6x+8}{x^2-x-6}$ (b) $\frac{3x^2-2x-8}{x^2-4}$

(c) $\frac{(x+3)^2-2(x+3)}{x^2+2x-3}$ (d) $\frac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

**MATHEMATICS A
AND MATHEMATICS B (MEI)****Student Guide****Exercise 1.5**

Solve the following simultaneous equations.

1 $x^2 + xy = 12$

$3x + y = 10$

2 $x^2 - 4x + y^2 = 21$

$y = 3x - 21$

3 $x^2 + xy + y^2 = 1$

$x + 2y = -1$

4 $x^2 - 2xy + y^2 = 1$

$y = 2x$

5 $c^2 + d^2 = 5$

$3c + 4d = 2$

6 $x + 2y = 15$

$xy = 28$

7 $2x^2 + 3xy + y^2 = 6$

$3x + 4y = 1$

8 $2x^2 + 4xy + 6y^2 = 4$

$2x + 3y = 1$

9 $4x^2 + y^2 = 17$

$2x + y = 5$

10 $2x^2 - 3xy + y^2 = 0$

$x + y = 9$

11 $x^2 + 3xy + 5y^2 = 15$

$x - y = 1$

12 $xy + x^2 + y^2 = 7$

$x - 3y = 5$

13 $x^2 + 3xy + 5y^2 = 5$

$x - 2y = 1$

14 $4x^2 - 4xy - 3y^2 = 20$

$2x - 3y = 10$

15 $x^2 - y^2 = 11$

$x - y = 11$

16 $\frac{12}{x} + \frac{1}{y} = 3$

$x + y = 7$

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

Exercise 1.6

1 Write the following as powers of x .

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n .

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x .

(a) $x^3\left(x + \frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x + \frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

7 Simplify $\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{100}+\sqrt{99}}$.

**MATHEMATICS A
AND MATHEMATICS B (MEI)****Student Guide****Exercise 2.1**

1 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1° .

(a) $\sin x^\circ = 0.9$ (b) $\cos x^\circ = 0.6$ (c) $\tan x^\circ = 2$

(d) $\sin x^\circ = -0.4$ (e) $\cos x^\circ = -0.5$ (f) $\tan x^\circ = -3$

2 Solve the following equations for $-180 \leq x < 180$. Give your answers to the nearest 0.1° .

(a) $\sin x^\circ = 0.9$ (b) $\cos x^\circ = 0.6$ (c) $\tan x^\circ = 2$

(d) $\sin x^\circ = -0.4$ (e) $\cos x^\circ = -0.5$ (f) $\tan x^\circ = -3$

3 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1° .

(a) $\sin 2x^\circ = 0.829$ (b) $\cos 3x^\circ = 0.454$ (c) $\tan 4x = 2.05$

(d) $\sin \frac{1}{2}x^\circ = 0.8$ (e) $\cos \frac{1}{2}x^\circ = 0.3$ (f) $\tan \frac{1}{3}x^\circ = 0.7$

Exercise 3.2

Sketch (do not plot) the general shape of the graphs of the following curves.

Axes are not required but can be included in the questions marked with an asterix.

1 $y = x^2 - 3x + 2$

2 $y = -x^2 + 5x + 1$

3 $y = 1 - x^2$

4 $y = (x - 2)(x + 4)$

5 $y = (3 - x)(2 + x)$

6 $y = (1 - x)(5 - x)$

7 $y = x^3$

8 $y = -x^3$

9* $y = \frac{3}{x}$

10* $y = -\frac{2}{x}$

11 $y = (x - 2)(x - 3)(x + 1)$

12* $y = \frac{2}{x^2}$

13 Sketch on the same axes the general shape of the graphs of $y = x^2$ and $y = x^4$.

14 Sketch on the same axes the general shape of the graphs of $y = x^3$ and $y = x^5$.

**MATHEMATICS A
AND MATHEMATICS B (MEI)****Student Guide****Exercise 3.3**

Sketch the curves in questions 1–21. Use a different diagram for each. Show the x -coordinates of the intersections with the x -axis.

1 $y = x^2$

2 $y = (x - 1)(x - 3)$

3 $y = (x + 2)(x - 4)$

4 $y = x(x - 3)$

5 $y = (x + 2)(3x - 2)$

6 $y = x(4x + 3)$

7 $y = -x(x - 3)$

8 $y = (2 - x)(x + 1)$

9 $y = (3 - x)(2 + x)$

10 $y = (x + 2)(x - 1)(x - 4)$

11 $y = x(x - 1)(x + 2)$

12 $y = -x(x - 1)(x + 2)$

13 $y = (3 - x)(2 - x)(1 - x)$

14 $y = (x - 1)^2(x - 3)$

15 $y = (x - 1)(x - 3)^2$

16 $y = (x + 1)^3$

17 $y = (2 - x)(x + 1)^3$

18 $y = (x + 1)(x + 2)(x - 1)(x - 2)$

19 $y = -(x + 3)(x + 2)(x - 1)(x - 4)$

20 $y = (x - 2)^2(x + 2)^2$

21 $y = (x - 1)(x - 2)^2(x - 3)^3$

- 22** (a) Sketch the graph of $y = x^2$.
(b) Sketch $y = 2x^2$ on the same axes.
(c) Sketch $y = x^2 + 1$ on the same axes.

- 23** (a) Sketch the graph of $y = \sqrt{x}$.
(b) Sketch $y = 2\sqrt{x}$ on the same axes.

- 24** (a) Sketch the graph of $y = \frac{1}{x}$.
(b) Sketch $y = \frac{1}{x} + 1$ on the same axes.

**MATHEMATICS A
AND MATHEMATICS B (MEI)****Student Guide**

- 25 (a) Sketch the graph of $y = \frac{1}{x^2}$.
- (b) Sketch $y = \frac{2}{x^2}$ on the same axes.
- 26 (a) Sketch the graph of $y = x^3$.
- (b) Sketch $y = 2x^3$ on the same axes.
- 27 (a) Sketch the graph of $y = x^4$.
- (b) Sketch $y = 3x^4$ on the same axes.
- 28 (a) Sketch the graph of $y = x^3 - 4x$.
[Hint: It cuts the x -axis at -2 , 0 and 2 .]
- (b) Sketch $y = 2x^3 - 8x$ on the same axes.
- 29 (a) Sketch the graph of $y = x^4 - x^2$.
[Hint: It cuts the x -axis at 1 and -1 , and touches the axis at 0 .]
- (b) Sketch $y = -x^4 + x^2$ on the same axes.
- 30 Sketch, on separate axes, the following graphs. Show the x -coordinates of the intersections with the x -axis.
- (a) $y = 4 - x^2$
- (b) $y = (x - 2)(x + 1)$
- (c) $y = -(x - 2)(x + 1)$
- (d) $y = x(x + 4)$
- (e) $y = (x - 2)^2$
- (f) $y = -(x + 1)^2$
- (g) $y = (1 - x)(2 + x)$

Answers, hints and comments**Exercise 1.1**

1 (a) $p = 7$ (b) $q = 5$ (c) $r = 10$

2 (a) $x = 1$ (b) $x = 5$ (c) $x = -34$

3 $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

4 (a) $2(x + 5)$ (b) $4(x + 2)$ (c) $3(x + 7)$ (d) $2(x - 3)$

5 (a) $x = -4$ (b) $x = 9$ (c) $x = \frac{1}{18}$

6 (a) $x = \frac{2b - ad}{ac - b}$ (or $\frac{ad - 2b}{b - ac}$) (b) $x = \frac{d(4 - b)}{bc - 2}$ (or $x = \frac{d(b - 4)}{2 - bc}$)

7 (a) a (b) 4 (c) c^4 (d) $\frac{1}{4}d^4$

Exercise 1.2

1 (a) $\frac{20}{7}$ (b) $\frac{5}{4}$ (c) $\frac{14}{9}$ (d) $\frac{4}{5}$

(e) $\frac{2}{11}$ (f) $\frac{8}{33}$ (g) $\frac{2}{7}$ (h) $\frac{6}{35}$

(i) $\frac{3x^2}{y}$ (j) $\frac{3x}{y}$ (k) $\frac{5x^2}{4y}$ (l) $\frac{5x^2}{6y^2}$

(m) $\frac{15x^4}{2y}$ (n) $\frac{3y^4}{2xz}$ (o) $\frac{3xy^2}{5z}$ (p) $\frac{5a^2}{12x^3yz^2}$

2 (a) $x = \sqrt{\frac{A}{2\pi}}$ (b) $x = \sqrt[3]{\frac{3V}{4\pi}}$ (c) $x = \frac{u + v}{2t}$ (d) $x = \sqrt{\frac{3W}{2\pi h}}$

3 (a) $\frac{1+x}{1+3x}$ (b) $\frac{2+x}{3-x}$ (c) $\frac{3+2x}{2+x}$

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

$$4 \quad (a) \quad \frac{3x+5}{(x-1)(x+3)} \quad (b) \quad \frac{x+7}{(x-3)(x+2)} \quad (c) \quad \frac{x+3}{(2x-1)(3x+2)}$$

$$(d) \quad \frac{x+5}{x+2} \quad (e) \quad \frac{2x-3}{x-1} \quad (f) \quad -\frac{x+3}{x+1}$$

$$(g) \quad \frac{6x-1}{4(2x-1)(2x+1)}$$

$$5 \quad (a) \quad \frac{2x+1}{\sqrt{x}} \quad (b) \quad \frac{3x+3}{\sqrt{x+3}} \quad (c) \quad \frac{2x-2}{\sqrt[3]{x-2}}$$

$$6 \quad (a) \quad 1 + \frac{6}{x-5} \quad (b) \quad 1 + \frac{2}{x+1} \quad (c) \quad 1 - \frac{3}{x+5} \quad (d) \quad 1 - \frac{4}{x-2}$$

$$7 \quad (a) \quad 1 = A(x+1) + B(x-2)$$

$$(b) \quad x+2 = A(x-3) + B(x+2)$$

$$(c) \quad 2 = A(x+2)(x-3) + B(x+1)(x-3) + C(x+1)(x+2)$$

To clear fractions in part (d) you multiply both sides by $(x-2)^2(x+1)$, *NOT* by $(x-2)^2(x-2)(x+1)$.

$$(d) \quad 1 = A(x-2)(x+1) + B(x+1) + C(x-2)^2$$

$$(e) \quad 1 = Ax(x+2) + B(x+2) + Cx^2 \quad [\text{again, NOT } x^3 \text{ anywhere}]$$

Exercise 1.3

$$1 \quad (a) \quad x^2 + 10x + 25 \quad (b) \quad x^2 - 8x + 16 \quad (c) \quad 4x^2 + 4x + 1$$

$$(d) \quad 9x^2 - 12x + 4 \quad (e) \quad x^2 - 4 \quad (f) \quad 9x^2 - 16$$

$$2 \quad (a) \quad x + y + 2 = 0$$

$$(b) \quad 5x + 2y = 0$$

$$(c) \quad x + y = 0$$

$$3 \quad (a) \quad \text{impossible} \quad (b) \quad (x-2) \quad (c) \quad \text{impossible}$$

$$(d) \quad \text{impossible} \quad (e) \quad \text{impossible} \quad (f) \quad x + y$$

$$4 \quad (a) \quad (x+4)^2 + 3 \quad (b) \quad (x-5)^2 - 2 \quad (c) \quad (x+1)^2 - 5$$

$$(d) \quad (x-2)^2 - 7 \quad (e) \quad (x-1\frac{1}{2})^2 - \frac{1}{4} \quad (f) \quad (x-2\frac{1}{2})^2 - 12\frac{1}{4}$$

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

- 5 (a) $3(x+1)^2 + 4$ (b) $5(x-2)^2 - 3$ (c) $2(x+2\frac{1}{2})^2 + \frac{1}{2}$
- 6 (a) $(2x+3)^2 + 5$ (b) $(3x-2)^2 - 5$ (c) $(4x+5)^2 - 3$
- 7 (a) $(x-5)(x+5)$ (b) $4(x-3)(x+3)$ (c) $(2x-3y^2)(2x+3y^2)$
- (d) $(3x-1)(x-2)$ (e) $(3x-2)(x-1)$ (f) $(3x+2)(2x-3)$
- (g) $(4x+5)(2x-3)$
- 8 (a) $x^2 + 2 + \frac{1}{x^2}$ (b) $x^2 - \frac{1}{x^2}$ (c) $x^2 - 1 - \frac{6}{x^2}$

Exercise 1.4

- 1 (a) $12x$ (b) $4x^2$
- (c) $2+3x$ (d) $12x$ (e) $3+y$
- (f) $3xy$ (g) $\frac{2x+5y}{4x+3y}$ (h) $\frac{x-2y}{3x-y}$
- (i) can't be simplified (j) $\frac{2}{3}$ (k) $\frac{y}{2}$
- (l) can't be simplified (m) -1 (n) -1
- 2 (a) $x = \frac{bpy}{aqz}$ (b) $x = \frac{4by^2}{3\pi aqz}$
- 3 (a) $\frac{6x}{abr^3}$ (b) $\frac{3h}{2br^3}$
- 4 [see Example 5]
- (a) $(x-3)^2(5x-14)$ (b) $(2x+1)^3(14x+5)$
- (c) $\frac{1}{2}(k+1)(k+2)$ (d) $\frac{1}{6}(k+1)(k+2)(2k+3)$
- 5 (a) $\frac{x+4}{x-3}$ (b) $\frac{3x+4}{x+2}$
- (c) $\frac{x+1}{x-1}$ (d) $\frac{x(2x-1)}{x-1}$
- (e) $\frac{-1}{x^2\sqrt{x^2+1}}$ (f) $\frac{x-2}{2x^2\sqrt{1-x}}$

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

$$(g) \frac{-1}{2x\sqrt{x}\sqrt{x+1}} \quad (h) \frac{3+2x}{3(1+x)}$$

Exercise 1.5

The answer to the question “why not?” in example 1 (page 16) is that $x^2 + y^2$ has no simple square root. In particular it is not $x + y$. [Remember that $(x + y)^2 = x^2 + 2xy + y^2$.]

- | | | | |
|----|--|----|--|
| 1 | (2, 4), (3, 1) | 2 | (6, -3), (7, 0) |
| 3 | (1, -1), (-1, 0) | 4 | (1, 2), (-1, -2) |
| 5 | (2, -1), $(-\frac{38}{25}, \frac{41}{25})$ | 6 | (7, 4), $(8, \frac{7}{2})$ |
| 7 | (-5, 4), $(\frac{19}{5}, -\frac{13}{5})$ | 8 | (-1, 1), $(\frac{5}{3}, -\frac{7}{9})$ |
| 9 | (2, 1), $(\frac{1}{2}, 4)$ | 10 | (3, 6), $(\frac{9}{2}, \frac{9}{2})$ |
| 11 | (2, 1), $(-\frac{5}{9}, -\frac{14}{9})$ | 12 | (-1, -2), $(\frac{38}{13}, -\frac{9}{13})$ |
| 13 | $(\frac{5}{3}, \frac{1}{3}), (-\frac{3}{5}, -\frac{4}{5})$ | 14 | (2, -2) (only) |
| 15 | (6, -5) (only) | 16 | (6, 1), $(\frac{14}{3}, \frac{7}{3})$ |

Exercise 1.6

- 1 (a) x^{-1} (b) x^{-5} (c) $x^{1/5}$ (d) $x^{3/5}$ (e) $x^{-1/2}$ (f) $x^{-1/3}$
- 2 (a) $\frac{1}{x^4}$ (b) 1 (c) $\sqrt[6]{x}$ (d) $\sqrt[4]{x^3}$ (e) $\frac{1}{\sqrt{x^3}}$
- 3 (a) $4x^{1/3}$ (b) $3x^{-2}$ (c) $5x^{-1/2}$ (d) $\frac{1}{2}x^{-3}$ (e) $6x^0$
- 4 (a) $x^4 + x^2$ (b) $x^2 + x^{-2}$ (c) $x^{-4} + x^{-7}$
- 5 (a) $5\sqrt{3}$ (b) $6\sqrt{5}$ (c) $2\sqrt{6}$ (d) $\frac{1}{5}\sqrt{5}$ (e) $\frac{1}{2}\sqrt{3}$
- 6 (a) $\sqrt{2} + 1$ (b) $\sqrt{6} + 2$ (c) $2(\sqrt{7} - 2)$ (d) $\frac{1}{4}(3 - \sqrt{5})$ (e) $\sqrt{6} + \sqrt{5}$
- 7 In this question apply the method of 5(e) to each separate part.
- $$(\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \dots + (\sqrt{100} - \sqrt{99}) = \sqrt{100} - \sqrt{1} = 9.$$

Exercise 2.1

- 1 (a) 64.2, 115.8 (b) 53.1, 306.9 (c) 63.4, 243.4

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

- (d) 203.6, 336.4 (e) 120, 240 (f) 108.4, 288.4
- 2** (a) 64.2, 115.8 (b) 53.1, -53.1 (c) 63.4, -116.6
(d) -23.6, -156.4 (e) 120, -120 (f) -71.5, 108.4
- 3** (a) 28, 62, 208, 242 (b) 21, 99, 141, 219, 261, 339
(c) 16, 61, 106, 151, 196, 241, 286, 331 (d) 106.2, 253.7
(e) 145.1 (f) 105

Exercise 2.2

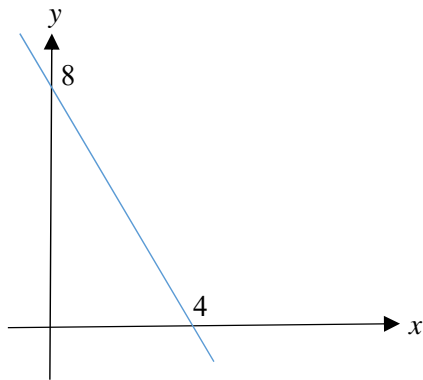
- 1** (a) $\frac{5}{13}, \frac{12}{5}$ (b) $\frac{6}{\sqrt{85}}, \frac{7}{\sqrt{85}}$ (c) $\frac{\sqrt{39}}{8}, \frac{\sqrt{39}}{5}$
- 2** (a) $p = 20 \sin 26^\circ, q = 20 \cos 26^\circ$ (b) $r = 5.6 \sin 32^\circ, s = 5.6 \cos 32^\circ$
(c) $t = 10 \sin 17^\circ, u = 10 \cos 17^\circ$ (d) $v = 8.4 \cos 20^\circ, w = 8.4 \sin 20^\circ$

Exercise 3.1

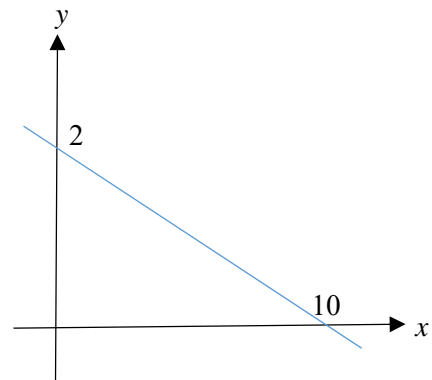
- 1** (a) $3x - y = 2$ (b) $x - 2y + 6 = 0$
(c) $3x + 4y = 12$ (d) $14x - 4y = 5$
(e) $8x + 12y = 9$ (f) $12x - 21y = 14$
- 2** (a) $y = -2x + 8; -2, 8$ (b) $y = 4x + 9; 4, 9$
(c) $y = -\frac{1}{5}x + 2; -\frac{1}{5}, 2$ (d) $y = \frac{1}{3}x - 5; \frac{1}{3}, -5$
(e) $y = -\frac{2}{3}x - 4; -\frac{2}{3}, -4$ (f) $y = \frac{5}{2}x - 10; \frac{5}{2}, -10$
(g) $y = -\frac{3}{5}x + \frac{17}{5}; -\frac{3}{5}, \frac{17}{5}$ (h) $y = \frac{7}{4}x + \frac{9}{2}; \frac{7}{4}, \frac{9}{2}$

AS and A LEVEL
MATHEMATICS A
AND MATHEMATICS B (MEI)
Student Guide

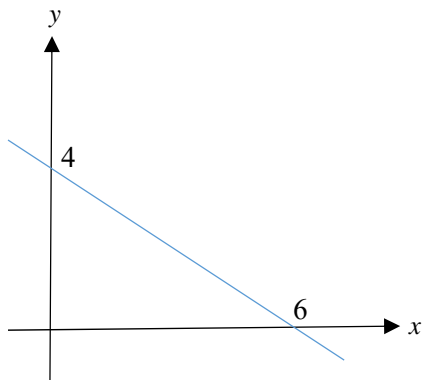
3 (a)



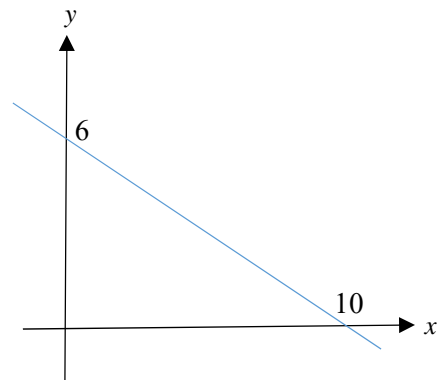
(b)



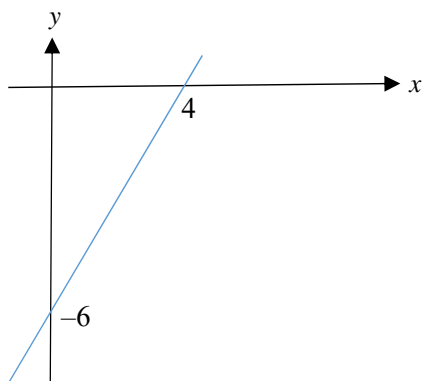
(c)



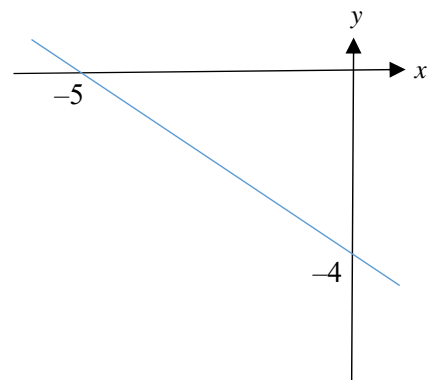
(d)



(e)



(f)



Exercise 3.2

1



2



3



4



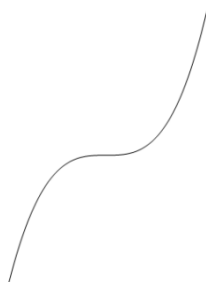
5



6



7

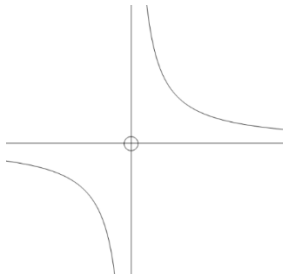


8

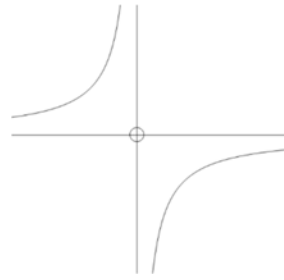


AS and A LEVEL
MATHEMATICS A
AND MATHEMATICS B (MEI)
 Student Guide

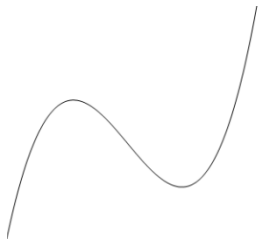
9



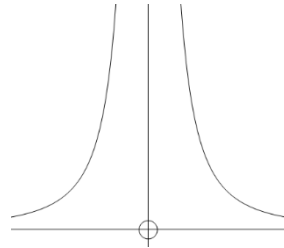
10



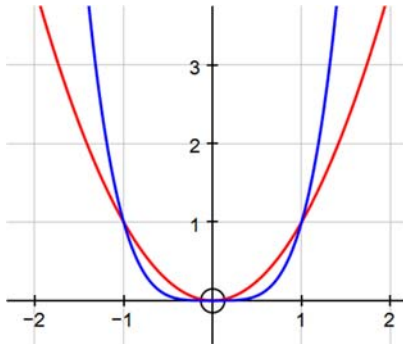
11



12

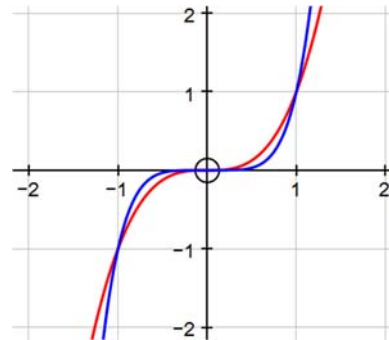


13



red: $y = x^2$ blue: $y = x^4$

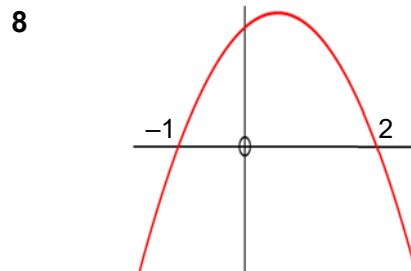
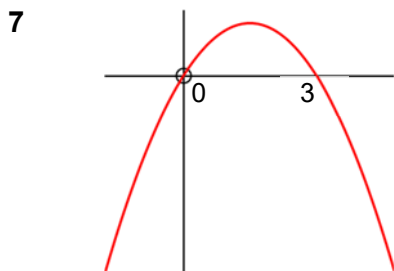
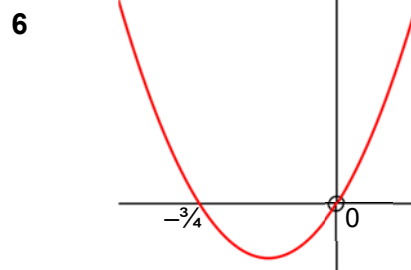
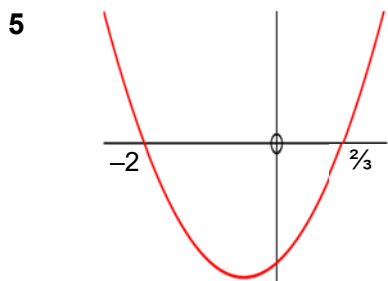
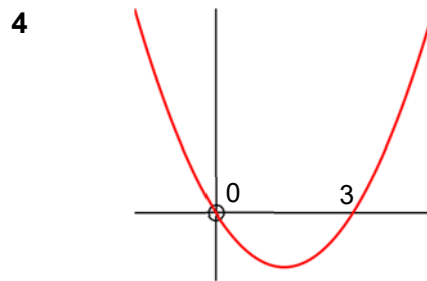
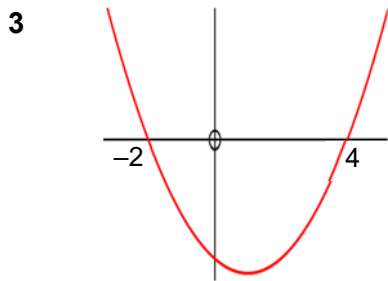
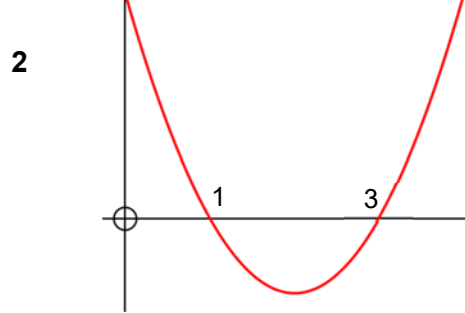
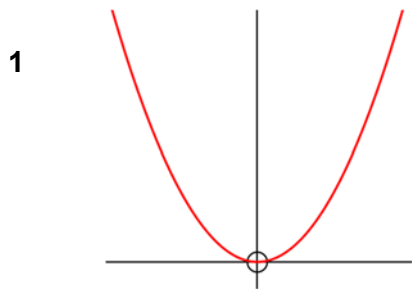
14



red: $y = x^3$ blue: $y = x^5$

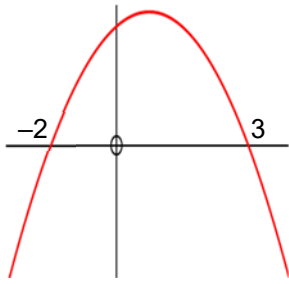
AS and A LEVEL
**MATHEMATICS A
AND MATHEMATICS B (MEI)**
Student Guide

Exercise 3.3

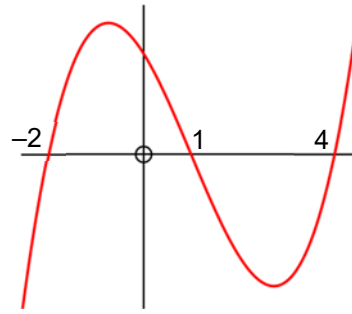


AS and A LEVEL
**MATHEMATICS A
AND MATHEMATICS B (MEI)**
Student Guide

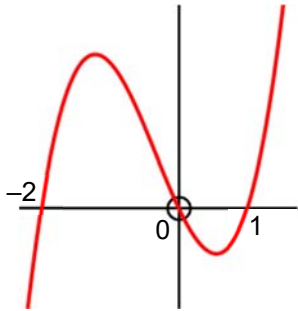
9



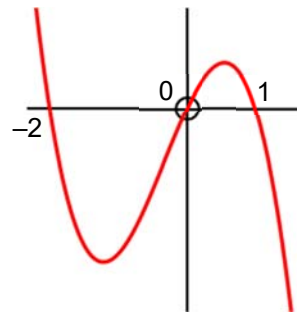
10



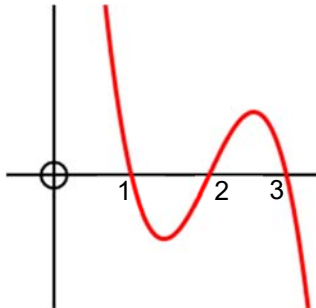
11



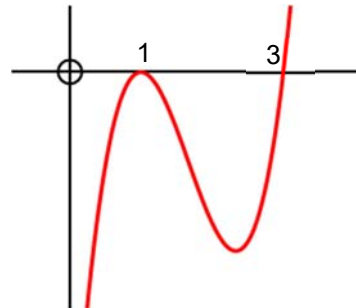
12



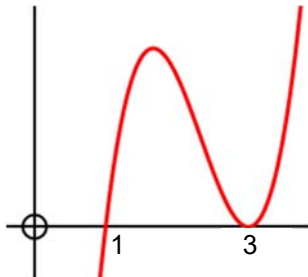
13



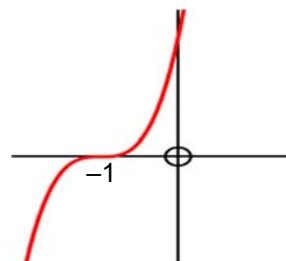
14



15

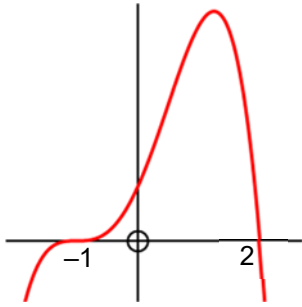


16

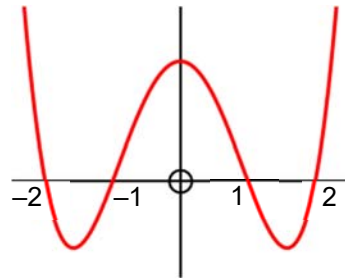


AS and A LEVEL
MATHEMATICS A
AND MATHEMATICS B (MEI)
 Student Guide

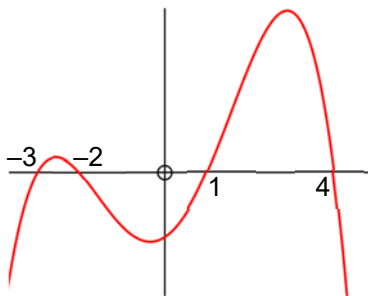
17



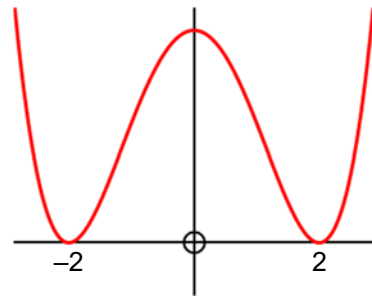
18



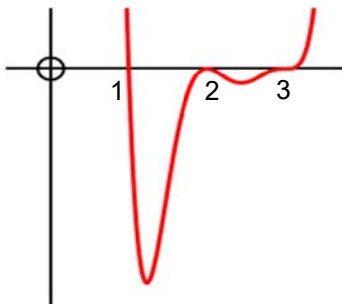
19



20

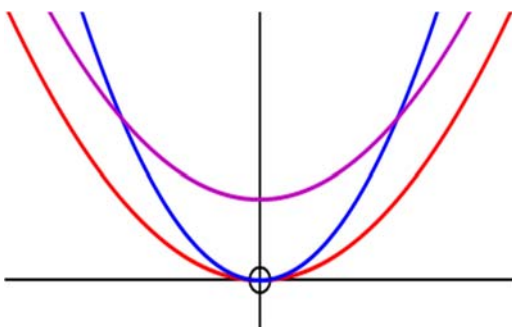


21



[In this graph in particular, do NOT worry about the y-coordinates of the minimum points.]

22



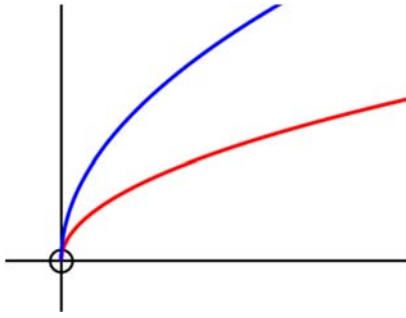
red: $y = x^2$

blue: $y = 2x^2$

purple: $y = x^2 + 1$

AS and A LEVEL
MATHEMATICS A
AND MATHEMATICS B (MEI)
 Student Guide

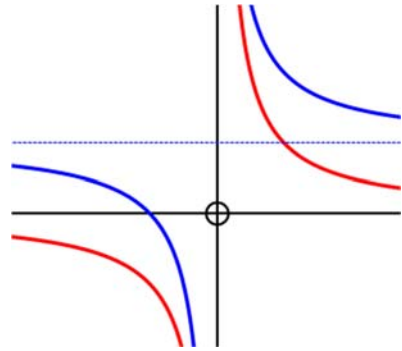
23



red: $y = \sqrt{x}$

blue: $y = 2\sqrt{x}$

24

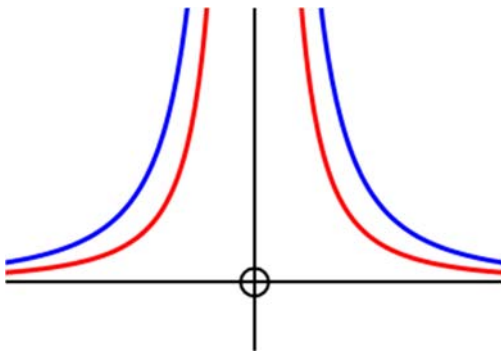


red: $y = \frac{1}{x}$

blue: $y = \frac{1}{x} + 1$

blue dotted: $y = 1$ [horizontal asymptote]

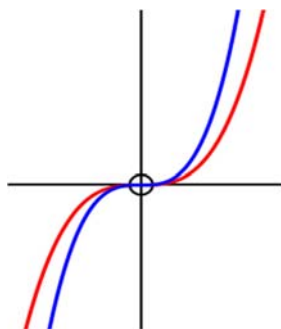
25



red: $y = \frac{1}{x^2}$

blue: $y = \frac{2}{x^2}$

26



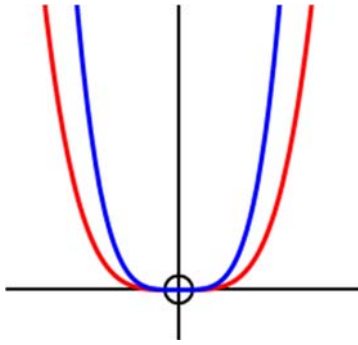
red: $y = x^3$

blue: $y = 2x^3$

MATHEMATICS A AND MATHEMATICS B (MEI)

Student Guide

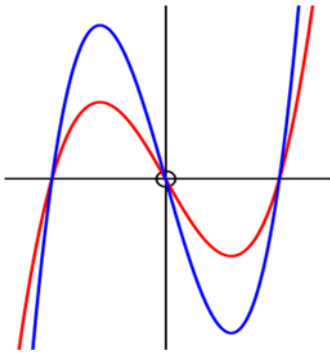
27



red: $y = x^4$

blue: $y = 3x^4$

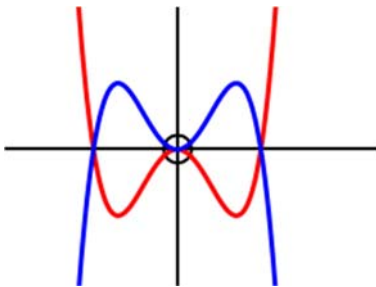
28



red: $y = x^3 - 4x$

blue: $y = 2x^3 - 8x$

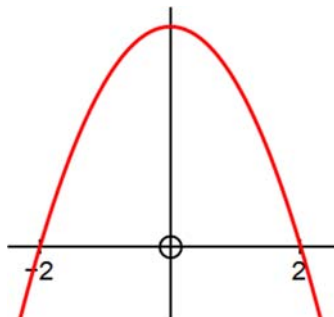
29



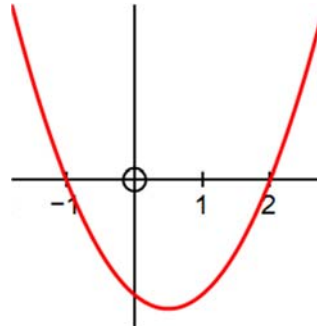
red: $y = x^4 - x^2$

blue: $y = -x^4 + x^2$

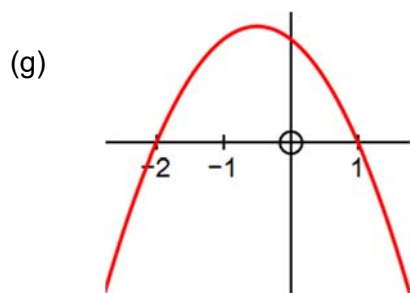
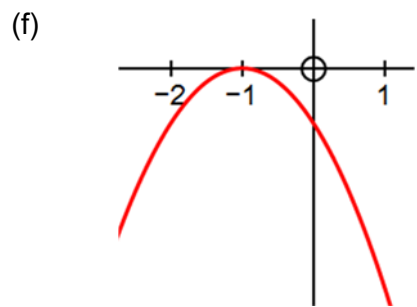
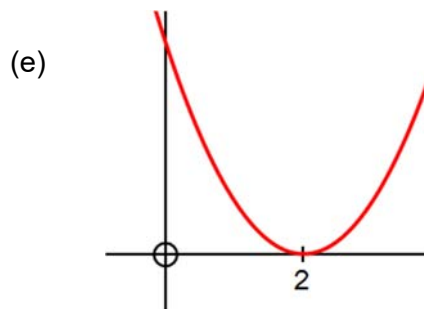
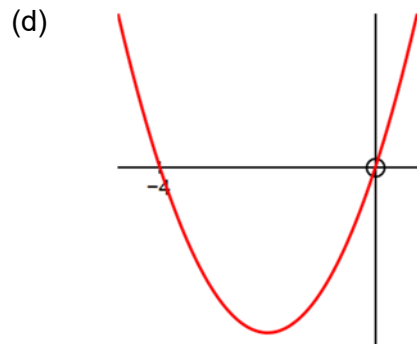
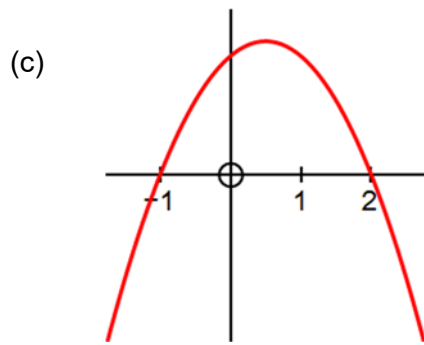
30 (a)



(b)



AS and A LEVEL
**MATHEMATICS A
AND MATHEMATICS B (MEI)**
Student Guide



NOTE: in parts (b), (c) and (g) in particular, the maximum or minimum point is *not* on the y-axis.